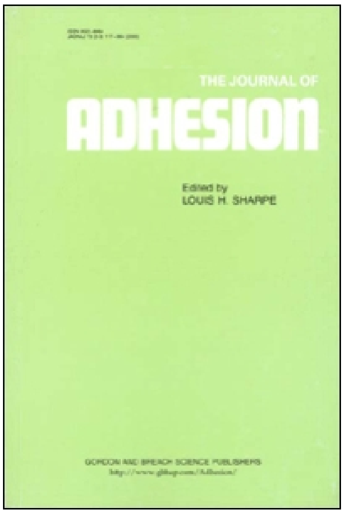

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Instability of a Large Coupled Microbeam Array Initialized at Its Two Ends

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A simple approximate method is suggested to determine the critical value for instability of a large parallel array of mutually attracting microbeams, based on instability analysis of a small array of only a few microbeams at the ends of the original large array. First, it is verified by a simplified spring system that equilibrium deflections of all intermediate microbeams (except those at the two ends of the parallel array) are negligibly small, and instability of the large microbeam array is initialized at the two ends of the array. Therefore, the critical value for instability of the original large array is determined by the critical value for instability of a small array of only a few microbeams at the two ends with its innermost microbeam fixed. The results obtained for the spring system show that the relative errors in the critical value between the original large array and the substitute small array are less than 2% when only three or four springs at each end are considered. In particular, the relative errors quickly converge to zero when the number of springs considered in the substitute small array further increases. This simple substitution method is used to approximately determine the critical value for instability of a large array of mutually attracting microbeams, and the results are compared with those obtained by other methods based on the instability analysis of the original large array, which contains a large number of microbeams. The present work offers a simple method to reduce the instability analysis of a large array of microbeams to a much simpler problem of a small array of only a few microbeams.

Keywords: MEMS; Microbeams; Pull-in instability; Structural instability; Surface forces

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1. INTRODUCTION

Typical microbeams in current microelectrical mechanical systems (MEMS) range from 0.1 to a few μm in thickness, with lateral dimensions of 10–500 μm and gaps between adjacent microbeams of around 1 μm [1]. Owing to drastic increase of surface area and decrease in thickness and gap, surface attractive forces, such as van der Waals force, electrostatic force, capillary force, and Casimir force [1–8], which are usually negligible in conventional structures at the macroscale, play a dominant role in mechanical deformation of microbeams and could lead to the unwanted jump-together instability of adjacent microbeams. Therefore, surface-force-driven structural instability and adhesion of microbeams have become a research topic of central importance in MEMS.

Parallel arrays of microbeams are commonly adopted in many designs of MEMS, especially in the comb-drive technology [9–13]. Exact structural instability analysis of such large arrays of mutually attracting microbeams raises a challenging nonlinear problem, especially when the number of parallel microbeams is large. This could explain why little effort has been made in the literature to study structural instability of a large array of mutually attracting microbeams. Very recently, we have developed a method for instability of a large parallel array of mutually attracting microbeams, based on the concept of the end effect on instability [14,15]. However, the method suggested in Refs. 14 and 15 is based on a simplified analysis of the original large microbeam array, which contains a large number of microbeams, and still suffers some technical complexity when the number of microbeams is extremely large. Therefore, it is of practical interest to develop an even easier method for the same problem.

In the present work, a simpler method is proposed to study instability of a large array of mutually attracting microbeams. This method is based on an observation, shown by our previous works [14,15], that equilibrium deflections of all intermediate beams (except those at the two ends of the large array) are negligibly small because two interactions from two adjacent beams on the opposite sides are almost equal but opposite and thus cancel each other. This implies that structural instability of the original large microbeam array is initialized at its two ends. Thus, it is anticipated that structural instability of the original large array of microbeams will be determined by structural instability of a small array of only a few outermost microbeams at each of the two ends. In this article, this idea is explored to develop a simple substitution method based on instability analysis of a small array of only a few microbeams. In Section 2, surface-force-driven instability of a large microbeam array is formulated. The effectiveness of the simple substitution method is examined in Section 3 for the spring system. In Sections 4 and 5, instability of a large array of identical microbeams or opposing microcantilevers is analyzed using this substitution method, with comparison to the results obtained in the previous works [14,15]. Finally, all results are summarized in Section 6.

2. FORMULATION OF INSTABILITY OF A LARGE ARRAY OF MICROBEAMS

Let us consider N microbeams, each of which attracts with two neighboring ones through surface forces, as shown in Figure 1. The cases considered include a large parallel array of identical microbeams of the bending rigidity EI and length L shown in Figure 1a–c or comb-drive microcantilevers consisting of two opposing parallel arrays of microcantilevers shown in Figure 1d where all cantilevers on the left side, with the bending rigidity E_1I_1 and length L_1, are labeled by odd subscripts (1, 3, 5,...) and defined by the axial coordinate x_1, and all cantilevers on the right side, with the bending rigidity E_2I_2 and length L_2, are labeled by even subscripts (2, 4, 6,...) and defined by the axial coordinate x_2, δ is the overlap depth of the opposing microcantilevers as shown in Figure 1.

As described in the existing literature [1–8], the attractive force per unit area between two surfaces of any two adjacent microbeams at any point can be given by $F = c/d^n$, where c is a constant depending on the nature of the interacting force and the materials, d is the distance between the two surfaces at that point, and the index n can be 2 (such as electrostatic force), 3 (such as unretarded van der Waals force), or 4 (such as Casimir force or retarded van der Waals force). Thus, the interacting force per unit axial length between any two adjacent beams is given by $f = Fb = C/d^n$, where $C = cb$ and b is the width of the interacting area of two adjacent microbeams as shown in Figure 1. In the present article, we consider only the case in which one type of the surface forces is dominant over the others, and thus $n = 2$, 3, or 4. In other words, combined effects of more than one type of the surface forces will not be examined in the present article. In addition, the distance between the microbeams and other possible surrounding materials is assumed to be so large that the associated interaction forces are negligible as compared with the beam–beam interaction. Here, the width b and the length L are assumed to be much larger than the gap d so that the nonuniform interaction effect (such as

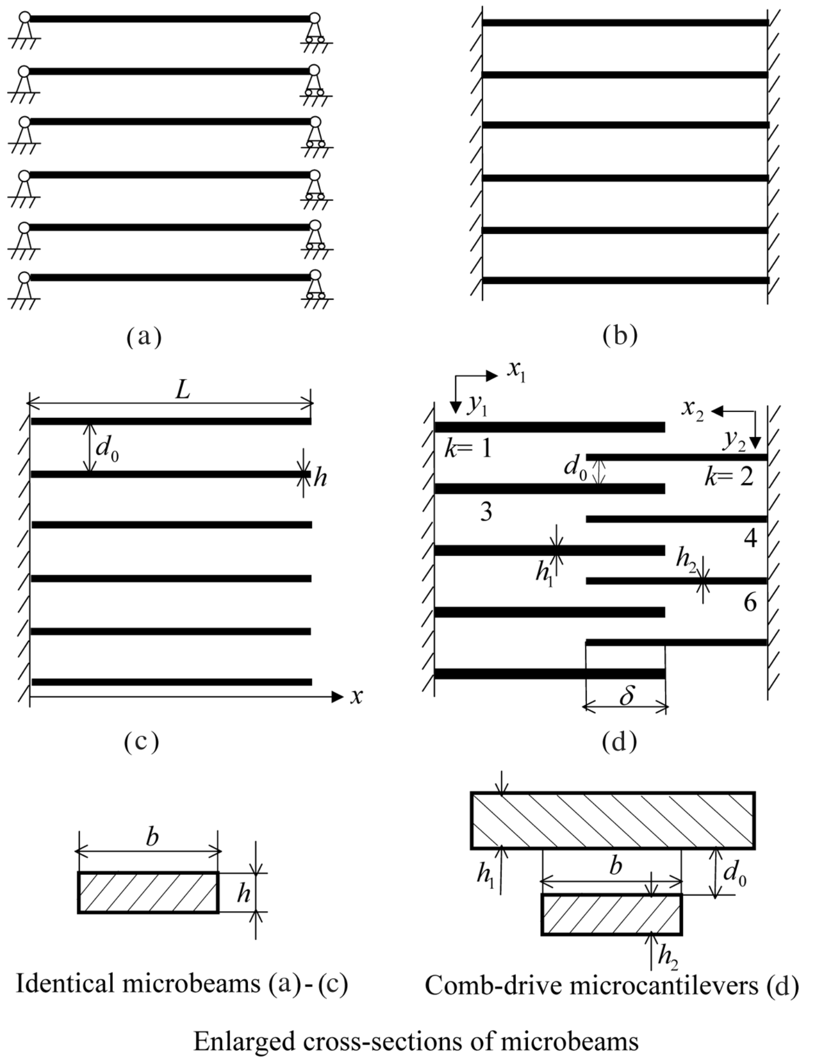

FIGURE 1 Large array of mutually attracting microbeams: a) hinged, b) clamped, c) cantilever, and d) comb-drive.

fringing field; see p. 1068 in [16] is negligible, and then the large parallel plate model for uniform interaction described previously works well for the beam–beam interaction studied here.

Assume that $Y_k(x)$ be the equilibrium deflection of beam k ($k = 1, 2, \ldots, N$) defined downward positive. For a parallel array of identical microbeams shown in Figure 1a–c, we have

$$EI\frac{d^4Y_k}{dx^4} = P_k \tag{1}$$

where P_k is the resultant force per unit axial length acting on beam k due to the interactions with two adjacent beams $(k-1)$ and $(k+1)$ given by

$$P_k = \frac{-C}{(d_0 + Y_k - Y_{k-1})^n} + \frac{C}{(d_0 + Y_{k+1} - Y_k)^n} \tag{2}$$

where P_k is defined positive along the deflection direction and d_0 is the initial separation between the two flat surfaces of any two adjacent beams.

For comb-drive microcantilevers (see Figure 1d), we have

$$E_1I_1\frac{d^4Y_k(x_1)}{dx_1^4} = P_k, \qquad \text{for } k = 1, 3, 5, \ldots \tag{3}$$

$$E_2I_2\frac{d^4Y_k(x_2)}{dx_1^4} = P_k, \qquad \text{for } k = 2, 4, 6, \ldots, \tag{4}$$

where P_k is the resultant force per unit axial length acting on cantilever k due to the interactions with two adjacent cantilevers $(k-1)$ and $(k+1)$, which vanishes outside the overlap domain and is given by

$$P_k = \frac{-C}{(d_0 + Y_k - Y_{k-1})^n} + \frac{C}{(d_0 + Y_{k+1} - Y_k)^n} \quad \text{for } L_1 - \delta \le x_1 \le L_1 \text{ or } L_2 - \delta \le x_2 \le L_2 \tag{5}$$

within the overlap domain.

Structural instability of N mutually interacting beams can be studied by the equilibrium method [17,18]. The instability of the beam array is defined by a critical value of the beam–beam interaction beyond which some adjacent microbeams jump together (called "adhesion") so that the distance reduction between them is larger than the initial separation d_0. Therefore, instability analysis of the large microbeam array requires studying the dependence of the equilibrium deflections of the microbeams, $Y_k(x)$ ($k = 1, 2, \ldots, N$), on the beam–beam interaction, which raises a challenging nonlinear problem especially when the number (N) of the microbeams is large.

As shown by our previous works [14,15], equilibrium deflections of all intermediate beams could be negligibly small because two interactions from two adjacent beams on the opposite sides are almost equal but opposite and thus cancel each other. This means that structural instability of a large array of microbeams will be initialized at the ends of the large array and characterized by collision of adjacent microbeams at its two ends. Therefore, it is expected that instability of the original large array can be determined by instability of a small array of only a few (3, 4, or more) microbeams at each of the two ends with the innermost microbeam fixed. Based on this idea, the critical value of interaction coefficient for instability of the original large array can be well estimated by the critical value of interaction coefficient for instability of the substitute small array. In the next section, this idea is explored to study instability of the N-spring system.

3. INSTABILITY OF A SIMPLE SPRING SYSTEM

To illustrate the present substitution method, let us first consider N equally spaced and mutually attracting springs, arranged along a straight line, from $k = 1$ (left end) to $k = N$ (right end), as shown in Figure 2. For this spring system to represent not only the cases Figure 1a–c but also the case Figure 1d, we assume that all springs of odd index ($k = 1, 3, 5, \ldots$) have the spring constant q_1, whereas all springs of even index ($k = 2, 4, 6, \ldots$) have the spring constant q_2, and any two adjacent springs are attracted to each other through the force $f = M/d^n$, where M is a constant and d is the distance between the two springs.

Under the spring–spring interaction forces, all springs will displace from their original neutral positions. We assume the displacement of the kth spring to be Y_k (defined right positive in Figure 2), thus equilibrium of the N mutually attracting springs are governed by N dimensionless nonlinear equations for N unknowns $Y_k/d_0 (k = 1, 2, \ldots, N)$ given by

$$-\frac{Y_1}{d_0} + \frac{B}{(1 + (Y_2/d_0) - (Y_1/d_0))^n} = 0, \qquad k = 1, \tag{6a}$$

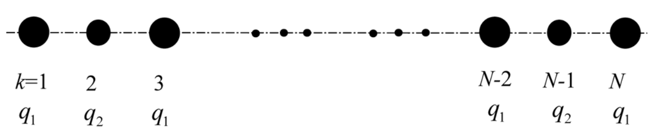

FIGURE 2 Spring system consisting of alternating array of springs of spring constant q_1 and springs of spring constant q_2.

$$-\frac{Y_k}{d_0}+\frac{\beta B}{(1+(Y_{k+1}/d_0)-(Y_k/d_0))^n}-\frac{\beta B}{(1+(Y_k/d_0)-(Y_{k-1}/d_0))^n}=0,\quad k=2,4,\ldots \tag{6b}$$

$$-\frac{Y_k}{d_0}+\frac{B}{(1+(Y_{k+1}/d_0)-(Y_k/d_0))^n}-\frac{B}{(1+(Y_k/d_0)-(Y_{k-1}/d_0))^n}=0,\quad k=3,5,\ldots \tag{6c}$$

$$-\frac{Y_N}{d_0}-\frac{B}{(1+(Y_N/d_0)-(Y_{N-1}/d_0))^n}=0,\qquad k=N \text{ is an odd number}, \tag{6d}$$

$$-\frac{Y_N}{d_0}-\frac{\beta B}{(1+(Y_N/d_0)-(Y_{N-1}/d_0))^n}=0,\qquad k=N \text{ is an even number}, \tag{6e}$$

where the two constants B and β are defined by

$$B=\frac{M}{q_1 d_0^{n+1}},\qquad \beta=\frac{q_1}{q_2}. \tag{7}$$

In particular, for a given ratio β, the constant B is the interaction coefficient defined on the initial distance d_0 between any two adjacent springs, which represents the intensity of the interaction between neighboring springs. It should be noted that Equation (6b) is identical to (6c) and Equation (6d) is identical to (6e) when $\beta = 1$.

Equilibrium displacements of all springs governed by Equation (6) can be obtained by the Newton iteration method. Equilibrium displacements of the springs suffer discontinuity when the loading parameter B reaches a certain critical value. For B smaller than the critical value, equilibrium displacements vary smoothly with the parameter B. When the loading parameter B exceeds the critical value, equilibrium displacements of the springs obtained from Equation (6) suffer a jump and lead to collision of some adjacent springs because the distance reduction between them is larger than the initial gap d_0. The exact critical values of B for instability and the equilibrium displacement of the end springs and their neighboring springs at the onset of instability for $\beta = 0.2$, 1, and 5 and $n = 2$, 3, and 4 are shown in Table 1. For example, when $n = 2$, for $\beta = q_1/q_2 = 0.2$ or 1, our numerical results show that equilibrium positions of the spring system

TABLE 1 Critical Values of B for Instability and the Equilibrium Displacement Change between the End Spring and Its Neighbor at the Onset of Instability for $\beta = 0.2$, 1, and 5 and n = 2, 3, and 4

β	n	N	Exact critical value of B	$(Y_1 - Y_2)/d_0$	$(Y_{N-1} - Y_N)/d_0$
0.2	2	20	1/7.543	0.3115	0.0482
		21	1/7.543	0.3115	0.3115
	3	20	1/10.651	0.2333	0.0367
		21	1/10.651	0.2333	0.2333
	4	20	1/13.753	0.1869	0.0297
		21	1/13.753	0.1869	0.1869
1	2	20	1/11	0.2439	0.2439
		21	1/11	0.2439	0.2439
	3	20	1/15.8	0.1781	0.1781
		21	1/15.8	0.1781	0.1781
	4	20	1/20.6	0.1398	0.1398
		21	1/20.6	0.1398	0.1398
5	2	20	1/37.713	0.0482	0.3134
		21	1/28.1	0.1705	0.1705
	3	20	1/53.254	0.0367	0.2339
		21	1/41.3	0.1221	0.1221
	4	20	1/68.765	0.0297	0.1869
		21	1/54.4	0.1050	0.1050

are stable when the loading parameter B given by Equation (7) is less than the critical value 1/7.543 for $\beta = 0.2$ or 1/11 for $\beta = 1$, regardless of N as an even or odd number. On the other hand, for $q_1/q_2 = 5$, equilibrium positions of the spring system are stable when the loading parameter B is less than 1/37.713 when N is an even number or when B is less than 1/28.1 when N is an odd number. Here a significant difference exists between the critical value with an even number N and the critical value with an odd number N when $q_1/q_2 = 5$, because the rigidity of the springs of odd indexes (1, 3,...) is much larger than the rigidity of the springs of even indexes (2, 4,...), and therefore the spring system has a lower critical value of B for instability when N is an even number. Obviously, when N is an odd number, both the left and the right end springs ($k = 1$ and $k = N$) have the same much larger rigidity, and then the critical value of B for instability is much higher. In particular, no such a difference exists for even or odd number N when $\beta = 0.2$ because the left end spring $k = 1$ always has the smaller rigidity, which determines the instability of the spring system, regardless of N as an even or odd number.

Equilibrium displacements of the spring system prior to instability are shown in Figure 3 for a few examples. It is seen from Figure 3 that

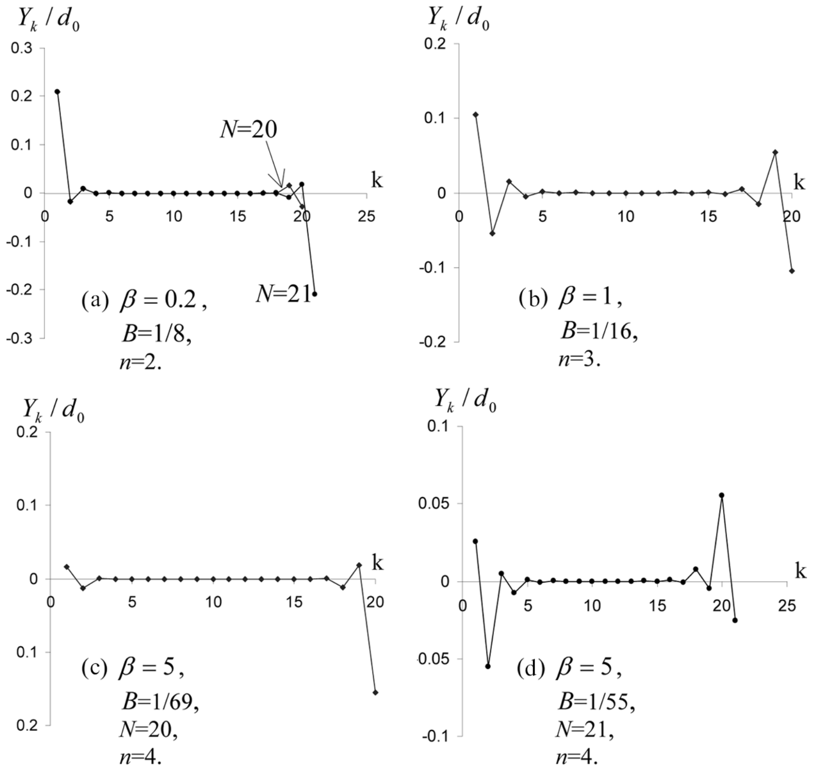

FIGURE 3 Equilibrium displacements of N mutually attracting springs governed by Equation (6) when $N = 20$ or 21 prior to instability, where $\beta = q_1/q_2$, and the loading parameter B is defined by Equation (7) based on the initial uniform separation d_0: a) $\beta = 0.2$, $B = 1/8$, and $n = 2$; b) $\beta = 1$, $B = 1/16$, and $n = 3$; c) $\beta = 5$, $B = 1/69$, $N = 20$, and $n = 4$; d) $\beta = 5$, $B = 1/55$, $N = 21$, and $n = 4$.

when the loading parameter B is less than the critical value, as expected, accurate equilibrium displacements of almost all intermediate springs are negligibly small. Actually, in all cases shown, the displacement of the spring $k = 3$ at the left end (or the spring $k = N - 2$ at the right end) prior to the onset of instability is negligibly small (around $0.02d_0$ or less), compared with the much larger displacements of the end springs ($k = 1$ and $k = N$) and their neighboring springs ($k = 2$ or $k = N - 1$). For instance, for $n = 2$, $B = 1/7.6$, $\beta = 0.2$, and $N = 20$, the displacement of the third spring is $0.014d_0$, and the displacements of the fourth and fifth springs are $-0.0009d_0$ and $0.0005d_0$, respectively. For $n = 3$, $B = 1/15.9$, $\beta = 1$, and $N = 20$ or 21, the displacement of the third spring is $0.017d_0$, and the displacements of the fourth and fifth springs are $-0.0060d_0$ and $0.0020d_0$, respectively. For $n = 4, B = 1/54.5, \beta = 5$, and $N = 21$, the displacement of the third spring is $0.0057d_0$, and the displacements of the fourth and fifth springs are $-0.0096d_0$ and $0.0009d_0$, respectively.

Based on the fact that equilibrium deflections of all intermediate springs (except those at the ends) are negligibly small, it is anticipated that structural instability of the original large spring array is determined by structural instability of only a few outermost springs (say N^* springs with $N^* \ll N$) at each of the two ends of the large array in which the deflection of the innermost spring $k = N^*$ (left end) or $K = (N - N^* + 1)$ (right end) can be assumed to be zero. Therefore, in doing so, the critical value for structural instability of the original large array (a much more complicated problem) can be reduced to determining the critical value for structural instability of a small array of only a few springs at the two ends (a much simpler problem).

It should be stated that which of the two ends determines instability of the original large array depends on the parameters β and N. If N is an even number, for $\beta < 1$, the deflection of the spring $k = 1$ is bigger than that of the spring $k = N$ because of the smaller rigidity of the spring $k = 1$. Thus, instability of the large array will be initialized on the left end, and then instability of the original large array depends on structural behavior of a few (N^*) springs at the left end with the innermost spring $k = N^*$ fixed. On the other hand, for $\beta > 1$, because the rigidity of the spring $k = N$ is smaller than that of the spring $k = 1$, instability of the original large array depends on structural behavior of a few (N^*) springs at the right end with the innermost spring $K = (N - N^* + 1)$ fixed. When $\beta = 1$, all springs are identical, and the spring system is symmetric about the two ends. Thus, structural instability of the array can be determined by a few springs on either of the two ends. Finally, if N is an odd number, because the spring system is symmetric about the two ends, regardless of whether β is greater than, equal to, or less than 1, structural instability of the large array can be determined by the behavior of a few springs on either of the two ends.

In what follows, let us consider structural instability of N^* springs at the left end with the innermost spring $k = N^*$ fixed. The critical interaction coefficients for instability of this small array of N^* springs for $\beta = 0.2$, 1, or 5 will be compared with the exact critical value for instability of the original large array for $\beta = 0.2$ (N is an even or odd number), 1 (N is an even or odd number), or 5 (N is an odd number), obtained with the Newton iteration method shown in Table 1. For the case in which $\beta = 5$ and N is an even number, we should consider structural instability of N^* springs at the right end with the innermost spring $K = (N - N^* + 1)$ fixed. However, this case (with $\beta = 5$ and N is an even number) is equivalent to the case in which $\beta = 1/5 = 0.2$ (with an even number N) if we define the right end spring as the first ($k = 1$) and the left end as the last ($k = N$). Therefore, structural instability for the case $\beta = 0.2$ is equivalent to structural instability of the case $\beta = 5$ when N is an even number. Thus, without loss of the generality, we focus on the instability of N^* springs at the left end of the original large array, for $\beta = 0.2$, 1, or 5, respectively.

3.1. Estimate of the Critical Value with $Y_2 = 0$ ($N^* = 2$)

As the simplest approximation, let us first consider only two springs ($N^* = 2$) at the left end, with the inner spring $k = N^* = 2$ fixed (that is $Y_2 = 0$). Based on Equation (6a) with $Y_2 = 0$, the deflection of the left end spring is determined by

$$-\frac{Y_1}{d_0} + \frac{B}{(1 - (Y_1/d_0))^n} = 0. \tag{8}$$

The critical value for instability of the small array of only two springs governed by Equation (8) is 1/6.75 for $n = 2$, 1/9.482 for $n = 3$, or 1/12.208 for $n = 4$. In addition, the distance change between the end spring and its neighbor at the onset of instability is $0.3333d_0$ for $n = 2$, $0.2477d_0$ for $n = 3$, or $0.1978d_0$ for $n = 4$. On the other hand, the exact critical value for instability of the original large array, obtained by the Newton iteration method, is shown in Table 1. Thus, the average relative error in the critical value for instability is 130% for $n = 2$, or 138% for $n = 3$, or 142% for $n = 4$. In addition, the average relative error in the distance change between the end spring and its neighbor at the onset of instability is 46% for $n = 2$, or 49% for $n = 3$, or 45% for $n = 4$. Therefore, for the original large array, considering only two springs at its left end will lead to unacceptably large errors in the critical value for instability and the distance change between the end spring and its neighbor at the onset of instability.

3.2. Estimate of the Critical Value with $Y_3 = 0$ ($N^* = 3$)

Next, let us consider three springs at the left end of the large array, with the innermost spring $k = N^* = 3$ fixed (that is, $Y_3 = 0$). It follows from Equation (9) that the deflections of the left end spring and the second spring are determined by

$$-\frac{Y_1}{d_0}+\frac{B}{(1+(Y_2/d_0)-(Y_1/d_0))^n}=0 \tag{9a}$$

$$-\frac{Y_2}{d_0}+\frac{\beta B}{(1-(Y_2/d_0))^n}-\frac{\beta B}{(1+(Y_2/d_0)-(Y_1/d_0))^n}=0. \tag{9b}$$

The critical values for instability of the small array of three springs governed by Equation (9) for $\beta = 0.2$, 1, or 5 and $n = 2$, 3, or 4, as well as the corresponding distance changes between the end spring and its neighbor at the onset of instability, are shown in Table 2. It is seen from Table 2 that considering the small array of only three springs at the left end of the large array offers an effective simple method to estimate the critical value for instability of the original large array, with relative errors of less than 2%. In addition, the average relative error in the distance change between the end spring and its neighbor at the onset of instability is 2.8% for $n = 2$, 5.5% for $n = 3$, or 4.7% for $n = 4$. Hence, considering only three springs at the end has already led to useful approximate results with small relative errors.

3.3. Estimate of the Critical Value with $Y_4 = 0$ ($N^* = 4$)

Further, let us consider $N^* = 4$ springs at the left end of the large array with $Y_4 = 0$. It follows from Equation (10) that the deflections

TABLE 2 Critical Values of B for Instability and the Distance Change between the End Spring and Its Neighbor at the Onset of Instability for $\beta = 0.2$, 1, and 5 and n = 2, 3, and 4 when N^* (=3) Springs at the Left End of the Original Array are Considered and the Displacement of the Third Spring is Assumed to be Zero ($Y_{N^*} = 0$)

β	n	Critical value of B	Relative error of B (%)	$(Y_1 - Y_2)/d_0$	Relative error of $(Y_1 - Y_2)/d_0$ (%)
0.2	2	1/7.523	0.266	0.3109	0.193
	3	1/10.613	0.358	0.2186	6.301
	4	1/13.701	0.380	0.1856	0.696
1	2	1/10.9	0.917	0.2312	5.207
	3	1/15.5	1.936	0.1870	4.997
	4	1/20.2	1.980	0.1432	2.432
5	2	1/27.6	1.812	0.1655	2.933
	3	1/40.7	1.474	0.1159	5.078
	4	1/53.6	1.493	0.0935	10.952

of the first, second, and third springs are determined by

$$-\frac{Y_1}{d_0}+\frac{B}{(1+(Y_2/d_0)-(Y_1/d_0))^n}=0 \tag{10a}$$

$$-\frac{Y_2}{d_0}+\frac{\beta B}{(1+(Y_3/d_0)-(Y_2/d_0))^n}-\frac{\beta B}{(1+(Y_2/d_0)-(Y_1/d_0))^n}=0 \tag{10b}$$

$$-\frac{Y_3}{d_0}+\frac{B}{(1-(Y_3/d_0))^n}-\frac{B}{(1+(Y_3/d_0)-(Y_2/d_0))^n}=0 \tag{10c}$$

Similarly, the critical values for instability and the distance changes between the end spring and its neighbor at the onset of instability for $\beta = 0.2$, 1, or 5 and $n = 2$, 3, or 4 are shown in Table 3. It is seen from Table 3 that the small array of four springs at the left end of the large array offers an almost accurate critical value for instability of the original large array, with relative errors of less than 0.5%. The average relative error in the distance change between the end spring and its neighbor at the onset of instability is 2.5% for $n = 2$, 1.1% for $n = 3$, or 3.5% for $n = 4$.

3.4. Accuracy of the Simple Method for the Spring System

Further numerical results are obtained for larger N^* (omitted here). The results show that the difference between the critical value for

TABLE 3 Critical Values of B for Instability and the Distance Change between the End Spring and Its Neighbor at the Onset of Instability for $\beta = 0.2$, 1, and 5 and n = 2, 3, and 4 when N^* (= 4) Springs at the Left End of the Original Array are Considered and the Displacement of the Fourth Spring is Assumed to be Zero ($Y_{N^*} = 0$)

β	n	Critical value of B	Relative error of B (%)	$(Y_1 - Y_2)/d_0$	Relative error of $(Y_1 - Y_2)/d_0$ (%)
0.2	2	1/7.542	0.013	0.3127	0.385
	3	1/10.650	0.009	0.2322	0.472
	4	1/13.751	0.015	0.1858	0.589
1	2	1/11	0	0.2398	1.681
	3	1/15.8	0	0.1742	2.190
	4	1/20.5	0.488	0.1460	4.435
5	2	1/28.1	0	0.1612	5.455
	3	1/41.2	0.243	0.1230	0.737
	4	1/54.3	0.184	0.0992	5.524

$N^* = 5$ and the critical value for $N^* = 4$ is always less than 0.5% for $\beta = 0.2$, 1, or 5, and $n = 2$, 3, or 4. The estimated critical values of B for $N^* = 2, 3, 4$, or 5 are shown in Figure 4. It is seen from Figure 4 that the relative errors quickly converge to zero when the number N^* of springs considered in the substitute small array increases beyond $N^* = 5$. For example, when $N^* = 5$, the relative error in the critical value of interaction coefficient obtained by this substitution method is less than 0.1% for $\beta = 0.2$, 1, or 5, and $n = 2$, 3, or 4. In addition, this simple substitution method is also good in predicting the distance change between the end spring and its neighbor at the onset of instability, with relative errors around 5% (for $N^* = 5$). When $\beta > 1$ and N is an even number, similarly, we can consider a few (N^*) springs at the right end with the innermost spring $K = (N - N^* + 1)$ fixed in order to predict the structural instability of the original large array.

In conclusion, for a simplified spring system, the substitution method suggested here can be used to determine approximately the critical value for instability of the large array, based on an analysis of a substitute small array of only a few springs at its two ends. Clearly, such a substitution method largely simplifies the instability analysis of the original large array of interacting springs.

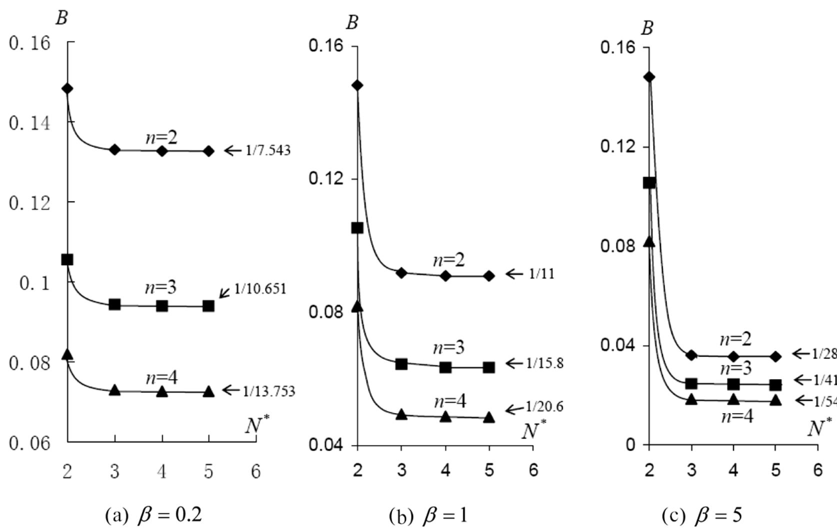

FIGURE 4 Critical values of B for instability of the small array of N^* springs at the left end of the original large spring array when $\beta = 0.2$, 1, or 5, and $n = 2$, 3, or 4, where the displacement of the N^*th spring is assumed to be zero ($Y_{N^*} = 0$).

4. INSTABILITY OF A PARALLEL ARRAY OF IDENTICAL MICROBEAMS

The results obtained for the spring system show that the proposed substitution method can provide reasonably accurate critical values of the interaction coefficient for instability of a original large array of interacting springs. In this section, this method is employed to determine the critical values for instability of a large parallel array of identical microbeams, as shown in Figure 1a–c.

4.1. Estimate of the Critical Value with $Y_2 = 0$ ($N^* = 2$)

First, let us consider two microbeams ($N^* = 2$) with the second microbeam fixed ($Y_2 = 0$). Based on Equations (1) and (2) with $Y_2 = 0$, the deflection of the end beam $Y_1(x)$ is determined by

$$EI\frac{d^4Y_1}{dx^4} = \frac{C}{(d_0 - Y_1)^n}. \tag{11}$$

Equation (11) can be solved numerically by the Galerkin method, and the deflection $Y_1(x)$ can be expressed by the first m fundamental modes of the beam as

$$Y_1(x) = \sum_{i=1}^{m} a_i F_i(x). \tag{12}$$

For hinged beams (Figure 1a), the first m fundamental modes have the simple form [17,18]

$$F_i(x) = \sin\left(\frac{i\pi x}{L}\right). \tag{13}$$

For clamped beams (Figure 1b), the first m fundamental modes are given by [17,18]

$$F_i(x) = \sin\beta_i x - \sinh\beta_i x - c_i(\cos\beta_i x - \cosh\beta_i x), \tag{14}$$

where $\beta_1 L = 4.730$, $\beta_2 L = 7.853$, $\beta_3 L = 10.996$, $\beta_4 L = 14.137$, $\beta_5 L = 17.279, \ldots$, and

$$c_i = \frac{\sin\beta_i L - \sinh\beta_i L}{\cos\beta_i L - \cosh\beta_i L}. \tag{15}$$

Equation (14) is also valid for cantilevers (Figure 1c), provided that [17,18] $\beta_1 L = 1.875$, $\beta_2 L = 4.694$, $\beta_3 L = 7.855$, $\beta_4 L = 10.996$,

$\beta_5 L = 14.137, \ldots,$ and

$$c_i = \frac{\sin \beta_i L + \sinh \beta_i L}{\cos \beta_i L + \cosh \beta_i L}. \tag{16}$$

Similar to previous papers [14,15], the interaction coefficient is defined based on the initial separation d_0 between adjacent microbeams as

$$A_0 = \frac{nC}{d_0^{n+1}}. \tag{17}$$

Multiplying Equation (11) by $F_i(x)$ ($i = 1, 2, \ldots, m$) and then integrating over $x = [0, L]$, one will obtain m equations with m unknown coefficients a_i ($i = 1, 2, \ldots, m$). The critical value of A_0 for instability of a large array of microbeams is determined as the lowest interaction coefficient at which the jump-together instability occurs. In what follows, we take $m = 3$ because the relative errors between $m = 3$ and $m = 4$ are already much less than 1%.

Because parallel arrays of microbeams in MEMS, especially in the comb-drive technology, are usually electrostatically controlled [9–13], we focus on the power index $n = 2$. For $n = 2$, the present method with $N^* = 2$ predicts that the critical value of $A_0/EI(\pi/L)^4$ for instability of the two microbeams is 0.283 for hinged beams, 1.449 for clamped beams, or 0.0345 for cantilevers. In addition, the maximum deflection of the end beam at the onset of instability, predicted by the present method ($N^* = 2$), is $0.3854d_0$ for hinged beams, or $0.3943d_0$ for clamped beams, or $0.4477d_0$ for cantilevers. On the other hand, the critical value of $A_0/EI(\pi/L)^4$ for $n = 2$ predicted by the method suggested in Ref. 14, based on instability analysis of the original large array of N microbeams, is 0.185 for hinged beams, 0.923 for clamped beams, or 0.0229 for cantilevers. Thus, the relative error in the critical value of the interaction coefficient for instability is 53% for hinged beams, 57% for clamped beams, or 51% for cantilevers. Therefore, for the large array of identical microbeams, considering only two microbeams at the ends will lead to large relative errors in the critical value of interaction coefficient for instability.

4.2. Estimate of the Critical Value with $Y_3 = 0$ ($N^* = 3$)

Let us further analyze instability of three microbeams ($N^* = 3$) at the end of the original large array shown in Figure 1a–c, with the third microbeam fixed ($Y_3 = 0$). It follows from Equation (18) that the deflections of the end beam and its neighbor are determined by

$$EI\frac{d^4Y_1}{dx^4} = \frac{C}{(d_0 + Y_2 - Y_1)^n}, \tag{18a}$$

$$EI\frac{d^4Y_2}{dx^4} = \frac{C}{(d_0 - Y_2)^n} + \frac{-C}{(d_0 + Y_2 - Y_1)^n}. \tag{18b}$$

With the Galerkin method, $Y_1(x)$ and $Y_2(x)$ can be expressed as

$$Y_1(x) = \sum_{i=1}^{m} a_i F_i(x),\ Y_2(x) = \sum_{i=1}^{m} b_i F_i(x). \tag{19}$$

Multiplying Equation (18) by $F_i(x)$ ($i = 1, 2, \ldots, m$) and then integrating over $x = [0, L]$, one obtains $2m$ equations with $2m$ unknown coefficients a_i and b_i ($i = 1, 2, \ldots, m$). Numerical results showed that when $n = 2$ and $N^* = 3$, the critical value of $A_0/EI(\pi/L)^4$ for instability of the three microbeams predicted by the present method is 0.178 for hinged beams, 0.915 for clamped beams, or 0.022 for cantilevers. In addition, the maximum distance change between the first and second beams at the onset of instability, predicted by the present method ($N^* = 3$), is $0.2820d_0$ for hinged beams, $0.3122d_0$ for clamped beams, or $0.3294d_0$ for cantilevers. Compared with the results given by the previous method in Ref. 14, the relative error in the critical value of the interaction coefficient for instability of the present method ($N^* = 3$) is 3.8% for hinged beams, 0.9% for clamped beams, or 3.9% for cantilevers. Thus, considering only three microbeams ($N^* = 3$) at the ends of the original large array can effectively estimate the critical value for instability of the original large array with relative errors of less than 4%.

4.3. Estimate of the Critical Value with $Y_4 = 0$ ($N^* = 4$)

Similarly, let us consider four microbeams ($N^* = 4$) at one end of the original array with the fourth microbeam fixed ($Y_4 = 0$). Thus, the deflections of the first, second, and third beams are determined by

$$EI\frac{d^4Y_1}{dx^4} = \frac{C}{(d_0 + Y_2 - Y_1)^n}, \tag{20a}$$

$$EI\frac{d^4Y_2}{dx^4} = \frac{C}{(d_0 + Y_3 - Y_2)^n} + \frac{-C}{(d_0 + Y_2 - Y_1)^n}, \tag{20b}$$

$$EI\frac{d^4Y_3}{dx^4} = \frac{C}{(d_0 - Y_3)^n} + \frac{-C}{(d_0 + Y_3 - Y_2)^n}. \tag{20c}$$

Using the Galerkin method, it is shown that when $n = 2$ and $N^* = 4$, the critical value of $A_0/EI(\pi/L)^4$ for instability of the four microbeams predicted by the present method is 0.176 for hinged beams, 0.904 for clamped beams, or 0.0218 for cantilevers. In addition, the maximum distance change between the first and second beams at the onset of instability is $0.2829d_0$ for hinged beams, $0.3058d_0$ for clamped beams, or $0.3358d_0$ for cantilevers. Thus, as compared with the results given by the previous method in Ref. 14, the relative error in the critical value of interaction coefficient for instability of the present method ($N^* = 4$) is 4.9% for hinged beams, 2.1% for clamped beams, or 4.8% for cantilevers. The critical values for instability given by the previous method in Ref. 14 are approximate in nature and cannot be used as the exact critical values. This can explain why the relative errors with $N^* = 4$ are even larger than the relative errors with $N^* = 3$. In fact, it is expected that the results given by the present substitution method quickly converge to the exact values when the number N^* increases.

The critical values, as a function of the number (N^*) of microbeams considered in the substitute small array,are shown in Figure 5, where

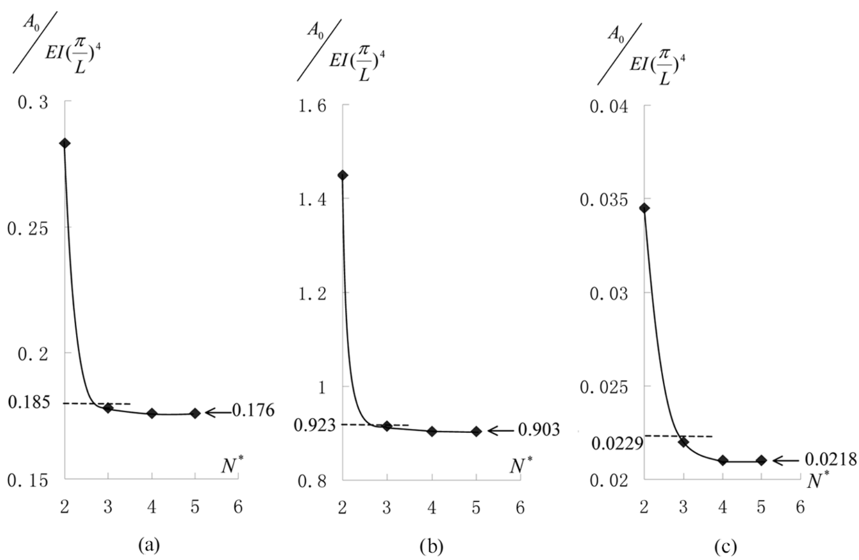

FIGURE 5 Critical value of the interaction coefficient for instability of a small array of N^* microbeams at the end of the original identical microbeam array when $n = 2$, where the N^*th microbeam is assumed to be fixed ($Y_{N^*} = 0$): a) hinged, b) clamped, c) cantilever.

the dashed lines represent the critical values for instability given by the previous approximate method in Ref. 14 for hinged, fixed beams, and cantilevers. Indeed, the difference between the critical value for instability with $N^* = 5$ and that with $N^* = 4$ is always less than 0.2% for hinged, fixed beams, or cantilevers. Because the critical value for instability decreases monotonically with increasing the number (N^*) of microbeams considered in the substitute small array, the present substitution method offers an accurate prediction in the critical value for instability of the large array when the number N^* increases.

5. INSTABILITY OF COMB-DRIVE MICROCANTILEVERS

Let us now use the simple substitution method to predict the critical value for instability of comb-drive microcantilevers, shown in Figure 1d. Because in almost all practical examples of comb-drive technology, such as those reported in Refs. 9–13, comb-drive microcantilevers have the same material and geometrical characteristics, we assume in this section that $E_1I_1 = E_2I_2 = EI$ and $L_1 = L_2 = L$.

5.1. Estimate of the Critical Value with $Y_2 = 0$ ($N^* = 2$)

Let us first consider two opposing microcantilevers ($N^* = 2$) with the second one fixed ($Y_2 = 0$). It follows from Equations (3–5) that the deflection $Y_1(x_1)$ of the first cantilever is determined by

$$\begin{aligned} EI\frac{d^4Y_1}{dx_1^4} &= 0 \qquad \text{for } 0 \le x_1 < L - \delta, \\ EI\frac{d^4Y_1}{dx_1^4} &= \frac{C}{(d_0 - Y_1)^n} \qquad \text{for } L - \delta \le x_1 \le L. \end{aligned} \tag{21}$$

With the Galerkin method, $Y_1(x_1)$ is expressed by the fundamental modes of the cantilever as

$$Y_1(x_1) = \sum_{i=1}^{m} a_i F_i(x_1), \tag{22}$$

where

$$F_i(x_1) = \sin\beta_i x_1 - \sinh\beta_i x_1 - c_i(\cos\beta_i x_1 - \cosh\beta_i x_1), \tag{23}$$

and the values of $\beta_i L$ and c_i are shown in Equation (16).

Multiplying Equation (21) by $F_i(x_1)$ ($i = 1, 2, \ldots, m$) and then integrating over $x_1 = [0, L]$, one can obtain m equations with m unknown coefficients a_i ($i = 1, 2, \ldots, m$). The critical values of $A_0/EI(\pi/L)^4$ given by the present method when $n = 2$ and $N^* = 2$ are shown in Figure 6 with respect to the overlap depth δ. It is seen from Figure 6 that the critical value of $A_0/EI(\pi/L)^4$ predicted by the present method with $N^* = 2$ is 0.0616 for $\delta/L = 0.2$, 0.0379 for $\delta/L = 0.5$, or 0.0346 for $\delta/L = 0.8$. In addition, the average distance change between the first cantilever and its neighbor at the onset of instability is 0.323 d_0 for $\delta/L = 0.2$, 0.273d_0 for $\delta/L = 0.5$, or 0.205d_0 for $\delta/L =$ 0.8. On the other hand, the critical values of the interaction coefficient for instability of the original array of comb-drive microcantilevers obtained

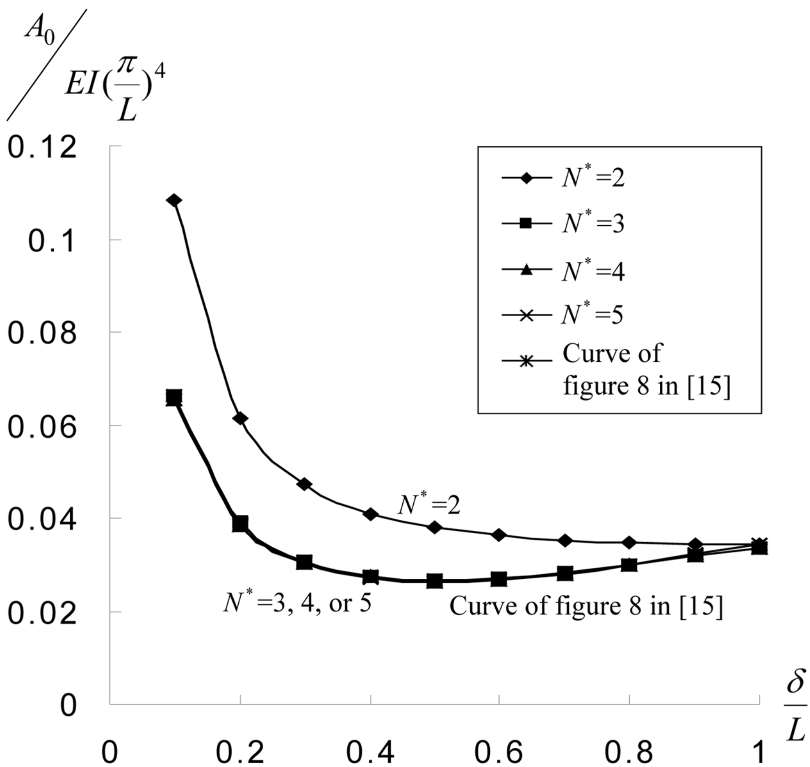

FIGURE 6 Critical value of the interaction coefficient for instability of a small array of N^* microcantilevers at the end of the original comb-drive microcantilever array when $n = 2$, as a function of the depth δ/L when $E_1I_1 = E_2I_2 = EI$, $L_1 = L_2 = L$, where the N^*th microcantilever is assumed to be fixed ($Y_{N^*} = 0$).

in our previous work (see Figure 8 of Ref. 15) is also shown in Figure 6. For example, the critical value in Figure 8 of Ref. 15 is 0.0389 for $\delta/L = 0.2$, or 0.0266 for $\delta/L = 0.5$, or 0.03 for $\delta/L = 0.8$. In addition, Grade *et al.* [11] found that for the comb-drive microcantilevers under electrostatic forces ($n = 2$), the maximum deflection of the end beams at the onset of instability is about $0.25d_0$. Thus, the present method ($N^* = 2$) leads to relative errors in the critical value of interaction coefficient for instability around 58% for $\delta/L = 0.2$, 42% for $\delta/L = 0.5$, or 15% for $\delta/L = 0.8$. The average relative error in the distance change between the first cantilever and its neighbor at the onset of instability is 19% for $\delta/L = 0.2$, 0.5, or 0.8. Obviously, for the large comb-drive microcantilever array, considering only two cantilevers at its ends leads to substantial errors.

5.2. Estimate of the Critical Value with $Y_3 = 0$ ($N^* = 3$)

Next, let us consider three cantilevers ($N = 3$) with the third one fixed ($Y_3 = 0$). It follows from Equations, (3–5) that the deflections of the end beam and its neighbor are determined by

$$\begin{aligned} EI\frac{d^4Y_1}{dx_1^4} &= 0 \qquad \text{for } 0 \le x_1 < L-\delta, \\ EI\frac{d^4Y_1}{dx_1^4} &= \frac{C}{(d_0+Y_2-Y_1)^n} \qquad \text{for } L-\delta \le x_1 \le L, \end{aligned} \tag{24a}$$

$$\begin{aligned} EI\frac{d^4Y_2}{dx_2^4} &= 0 \qquad \text{for } 0 \le x_2 < L-\delta, \\ EI\frac{d^4Y_2}{dx_2^4} &= \frac{C}{(d_0-Y_2)^n} + \frac{-C}{(d_0+Y_2-Y_1)^n} \qquad \text{for } L-\delta \le x_2 \le L. \end{aligned} \tag{24b}$$

With Galerkin's method, $Y_1(x_1)$ and $Y_2(x_2)$ can be expressed by

$$Y_1(x_1) = \sum_{i=1}^{m} a_i F_i(x_1), \qquad Y_2(x_2) = \sum_{i=1}^{m} b_i G_i(x_2), \tag{25}$$

where

$$G_i(x_2) = \sin\gamma_i x_2 - \sinh\gamma_i x_2 - c_i(\cos r_i x_2 - \cosh\gamma_i x_2) \tag{26}$$

and

$$\begin{aligned} &\gamma_1 L = 1.875, \quad \gamma_2 L = 4.694, \quad \gamma_3 L = 7.855, \quad \gamma_4 L = 10.996, \\ &\gamma_5 L = 14.137, \ldots. \end{aligned} \tag{27}$$

To solve Equation (24), one should change the variable x_2 in Equation (24a) into x_1 and change the variable x_1 in (24b) into x_2, using $x_1 + x_2 = 2L - \delta$. By multiplying Equation (24a) by $F_i(x_1)$ ($i = 1, 2, \ldots, m$) and then integrating over $x_1 = [0, L]$ and multiplying Equation (24b) by $G_i(x_2)$ ($i = 1, 2, \ldots, m$) and then integrating over $x_2 = [0, L]$, one will obtain $2m$ equations with $2m$ unknown coefficients a_i and b_i ($i = 1, 2, \ldots, m$). The critical values of $A_0/EI(\pi/L)^4$ given by the present method when $n = 2$ and $N^* = 3$ are shown in Figure 6, as a function of the overlap depth δ. It is seen from Figure 6 that this curve predicted by the present method with $N^* = 3$ is close to the curve of Figure 8 of Ref. 15. With the present method ($N^* = 3$), the critical value of $A_0/EI(\pi/L)^4$ for instability is 0.039 for $\delta/L = 0.2$, 0.0266 for $\delta/L = 0.5$, 0.5, or 0.03 for $\delta/L = 0.8$. In addition, the average distance change between the first cantilever and its neighbor at the onset of instability is $0.256d_0$ for $\delta/L = 0.2$, $0.249d_0$ for $\delta/L = 0.5$, or $0.242d_0$ for $\delta/L =$ 0.8. Thus, for $\delta/L = 0.2$, or 0.5, or 0.8, the relative error in the critical value of interaction coefficient for instability is less than 0.3%, and the relative error in the average distance change between the first cantilever and its neighbor at the onset of instability is less than 3.2%. Therefore, for the large comb-drive microcantilever array, considering only three microcantilevers ($N^* = 3$) at its ends of the large array gives a good estimate of the critical value for instability and the distance change between the first cantilever and its neighbor at the onset of instability, with reasonably small relative errors.

5.3. Estimate of the Critical Value with $Y_4 = 0$ ($N^* = 4$)

Further, if four cantilevers ($N^* = 4$) are considered with the fourth one fixed ($Y_4 = 0$), the deflections of the first, second, and third beams are determined by

$$EI\frac{d^4Y_1}{dx_1^4} = 0 \qquad \text{for } 0 \le x_1 < L - \delta,$$
$$EI\frac{d^4Y_1}{dx_1^4} = \frac{C}{(d_0 + Y_2 - Y_1)^n} \qquad \text{for } L - \delta \le x_1 \le L, \tag{28a}$$

$$EI\frac{d^4Y_2}{dx_2^4} = 0 \qquad \text{for } 0 \le x_2 < L - \delta,$$
$$EI\frac{d^4Y_2}{dx_2^4} = \frac{C}{(d_0 + Y_3 - Y_2)^n} + \frac{-C}{(d_0 + Y_2 - Y_1)^n} \qquad \text{for } L - \delta \le x_2 \le L, \tag{28b}$$

$$EI\frac{d^4Y_3}{dx_1^4} = 0 \qquad \text{for } 0 \leq x_1 < L - \delta,$$
$$EI\frac{d^4Y_3}{dx_1^4} = \frac{C}{(d_0 - Y_3)^n} + \frac{-C}{(d_0 + Y_3 - Y_2)^n} \qquad \text{for } L - \delta \leq x_1 \leq L, \tag{28c}$$

where $Y_1(x_1)$, $Y_2(x_2)$ and $Y_3(x_1)$ can be expressed by

$$Y_1(x_1) = \sum_{i=1}^{m} a_i F_i(x_1), Y_2(x_2) = \sum_{i=1}^{m} b_i G_i(x_2), Y_3(x_1) = \sum_{i=1}^{m} c_i F_i(x_1) \tag{29}$$

With the Galerkin method, the critical values of $A_0/EI(\pi/L)^4$ given by the present method when $n = 2$ and $N^* = 4$ are shown in Figure 6 with respect to the overlap depth δ. It is seen from Figure 6 that this curve is also close to the curve of Figure 8 of Ref. 15. With the present method ($N^* = 4$), the critical value of $A_0/EI(\pi/L)^4$ for instability is 0.0385 for $\delta/L = 0.2$, or 0.0263 for $\delta/L = 0.5$, or 0.0298 for $\delta/L = 0.8$. In addition, the average distance change between the first cantilever and its neighbor at the onset of instability is $0.253d_0$ for $\delta/L = 0.2$, $0.247d_0$ for $\delta/L = 0.5$, or $0.236d_0$ for $\delta/L =$ 0.8. Thus, compared to the results of Figure 8 of Ref. 15 for $\delta/L = 0.2$, 0.5, or 0.8, the relative error in the critical value for instability is less than 1.1%, and the relative error in the average distance change between the first cantilever and its neighbor at the onset of instability is less than 5.6%.

Furthermore, the critical values of $A_0/EI(\pi/L)^4$ for $N^* = 5$ are also shown in Figure 6, which are very close to those for $N^* = 4$. For example, with the present method ($N^* = 5$), the critical value of $A_0/EI(\pi/L)^4$ is 0.0384 for $\delta/L = 0.2$ (0.3% less than that for $N^* = 4$), 0.0263 for $\delta/L = 0.5$ (almost identical to that for $N^* = 4$), or 0.0298 for $\delta/L = 0.8$ (almost identical to that for $N^* = 4$). In addition, the average distance change between the first microcantilever and its neighbor at the onset of instability is $0.246d_0$ for $\delta/L = 0.2$, $0.252d_0$ for $\delta/L = 0.5$, or $0.238d_0$ for $\delta/L =$ 0.8. Thus, compared with the results of Figure 8 of Ref. 15 for $\delta/L = 0.2$, 0.5, or 0.8, the relative error in the critical value for instability is less than 1.3%, and the relative error in the average distance change between the first cantilever and its neighbor at the onset of instability is less than 4.8%.

To demonstrate the dependence of the predicted critical value on the number N^*, the critical values of $A_0/EI(\pi/L)^4$ given by the present method for $\delta/L = 0.2$, 0.5, or 0.8 are shown in Figure 7, as a function of the number N^*, where the dashed lines represent the corresponding critical values in the Figure 8 of Ref. 15. Here, it should be pointed

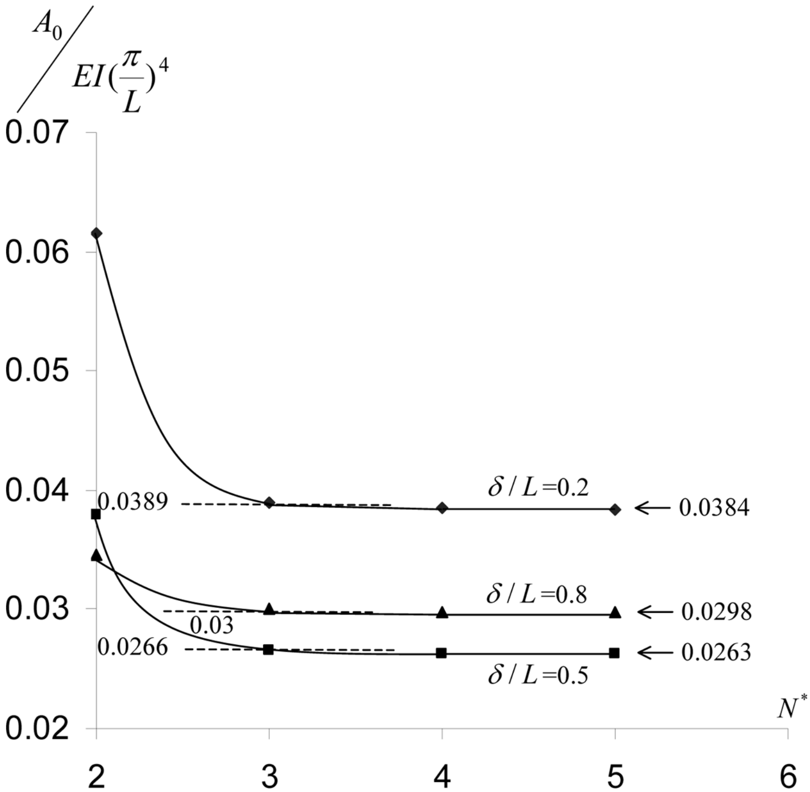

FIGURE 7 Critical value of the interaction coefficient for instability of a small array of N^* microcantilevers at the end of the original large comb-drive microcantilever array when $\delta/L = 0.2$, 0.5, or 0.8 and $n = 2$, where the N^*th microcantilever is assumed to be fixed ($Y_{N^*} = 0$).

out that the critical values for instability given by the previous method in Ref. 15 are approximate in nature and cannot be used as the exact critical values. This can explain why the relative errors with $N^* = 4$ are even larger than the relative errors with $N^* = 3$. In fact, it is expected that the results given by the present substitution method quickly converge to the exact values when the number N^* increases.

These results shown in Sections 4 and 5 show that the substitution method suggested here offers an alternative design criterion for structural stability of large arrays of microbeams, which is simpler than the procedure developed previously [14,15] and could be more easily applied to practical problems in MEMS. For example, when the interaction law $F = c/d^n$ or $f = C/d^n$ (see Section 2) is given, the results shown in Figures 5 and 7 give a critical separation d_0 below which the instability occurs. On the other hand, when the separation d_0 is fixed, the results shown in Figures 5 and 7 determine a critical value for the constant C determined by, for example, the Hamaker constant of the van der Waals forces or the applied electrical voltage for electrostatic forces, beyond which the instability occurs. Although vibration of a coupled array of microbeams has been recently studied [19,20] using a spring model with linearized interaction, structural instability of a large and nonlinearly coupled microbeam array and the associated end phenomena have not been studied in the literature at all.

In concluding our discussion, it should be stated that the present article is subject to a few limitations. First, the beam–beam interaction is restricted to only a single type of attractive forces ($n = 2$, 3, or 4) between the nearest adjacent beams; possible repulsive interaction (for example, between similarly charged adjacent microbeams) and combined interaction of more than one type of surface forces are not considered. Second, the so-called fringing field effect of electrostatic interaction has been neglected based on the present assumption that the gap between adjacent beams is small compared with other dimensions of the microbeams. We believe that the methods developed here can be extended to most of the more general cases without essential technical difficulties, although the extension to some cases would require a more complicated analysis. Also, the present substitution method is developed only for static instability of large coupled microbeam arrays but is not necessarily applicable to their nonlinear dynamics. To what degree the present ideas and methods can be extended to nonlinear dynamics of large coupled microbeam arrays requires a detailed separate study, which constitutes one interesting subject for future work.

Finally, the method suggested here is applicable not only to mutually attracting microbeams but also to nanobeams [21,22]. For instance, it has been well established that attractive van der Waals interactions between parallel carbon nanotubes often become the single dominant force in their mechanical deformation, and mechanical behavior of carbon nanotubes, as the most promising building blocks of future NEMS (nanoelectromechanical systems), can be well described by elastic beam models [23–27].

6. CONCLUSIONS

A simple substitution method is suggested to study structural instability of a large array of mutually attracting microbeams, based on instability analysis of a small array of only a few microbeams at the ends of the original large array with its innermost microbeam fixed. An exact analysis of the N-spring system confirms the accuracy of this simple substitution method. Further, this simple method is used to study instability of a large array of identical microbeams and comb-drive microcantilevers. Our results show that the present substitution method can predict the critical value for instability of the original large array of microbeams with reasonable relative errors (typically less than 5%), even when a small array of only four or five microbeams at the ends of the original large array are considered. In addition, the present method also predicts the distance change between the end microbeam and its neighbor at the onset of instability for the original large array. This simple substitution method reveals an essential feature of the instability of a large coupled microbeam array initialized at its two ends and offers a useful approximate criterion for structural instability of a large parallel array of mutually attracting microbeams or nanobeams in MEMS or NEMS.

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